Grade 6 Math Circles
October 4/5/6, 2022
Number Systems Solutions

Exercise Solutions

Exercise 1

Rewrite each expression as a power.

a) $8 \times 8 \times 8 \times 8 \times 8 \times 8$ b) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$ c) $5 \times 5 \times 5$

d) $7 \times 7 \times 7 \times 7 \times 7$ e) $10 \times 10 \times 10 \times 10$

Exercise 1 Solution

a) 8^6 b) 1^8 c) 5^3 d) 7^5 e) 10^4

Exercise 2

Rewrite each number as a power. Do not write a power with the exponent 1. There might be more than a single base possible for some numbers.

a) 49 b) 16 c) 1000 d) 1 e) 25

Exercise 2 Solution

a) $\begin{aligned} 49 &= 7 \times 7 \\ &= 7^2 \end{aligned}$

b) $\begin{aligned} 16 &= 4 \times 4 \\ &= 4^2 \end{aligned}$ or $\begin{aligned} 16 &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \end{aligned}$

c) $\begin{aligned} 1000 &= 10 \times 10 \times 10 \\ &= 10^3 \end{aligned}$

d) a^0 where a is any nonzero integer, or 1^n where n is any positive integer

e) $\begin{aligned} 25 &= 5 \times 5 \\ &= 5^2 \end{aligned}$

Exercise 3

Evaluate the following powers.

a) 2^3 b) 9^1 c) 6^2 d) 18^0 e) 10^6 f) 3^3

Exercise 3 Solution

a) 8 b) 9 c) 36 d) 1 e) 1000000 f) 27

Exercise 4

List the symbols used for each positional number system, in increasing order of their equivalent decimal system values.

a) Base 2 (binary) b) Base 8 (octal) c) Base 16 (hexadecimal)

Exercise 4 Solution

a) 0, 1 b) 0, 1, 2, 3, 4, 5, 6, 7

c) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. The letters A, B, C, D, E, F have values in the decimal system: 10, 11, 12, 13, 14, and 15, respectively.

Stop and Think

Why are letters used in hexadecimal instead of their corresponding decimal values?

Exercise 5

In Exercise 4, a standard convention was introduced as to what symbols are used in certain number systems. Write the following numbers using the proper notation for three number systems to which they could belong.

a) 3729 b) 104102 c) $32EJ9$

Exercise 5 Solution

Responses may vary. Below are some possible responses.

a) $3729_{10}, 3729_{16}, 3729_{100}$

b) $104102_5, 104102_9, 104102_{10}$

c) $32EJ9_{20}, 32EJ9_{23}, 32EJ9_{59}$

Exercise 6

Convert the following to decimal.

a) 1_2 b) 10_2 c) 11101_2

Exercise 6 Solution

a)
$$\begin{aligned} 1_2 &= 1_2 \times 2^0 \\ &= 1 \times 1 \\ &= 1_{10} \end{aligned}$$

b)
$$\begin{aligned} 10_2 &= (1_2 \times 2^1) + (0_2 \times 2^0) \\ &= (1 \times 2) + (0 \times 1) \\ &= 2 + 0 \\ &= 2_{10} \end{aligned}$$

c)
$$\begin{aligned} 11101_2 &= (1_2 \times 2^4) + (1_2 \times 2^3) + (1_2 \times 2^2) + (0_2 \times 2^1) + (1_2 \times 2^0) \\ &= (1 \times 16) + (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1) \\ &= 16 + 8 + 4 + 0 + 1 \\ &= 29_{10} \end{aligned}$$

Exercise 7

Convert the following to binary. Use the conversion table from Example 7.

a) 17_{10} b) 10_{10} c) 0_{10}

Exercise 7 Solution

a)

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
17			|				|	10001
	0	0	1	0	0	0	1	

Therefore, $17_{10} = 10001_2$.

b)

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
10				|		|		1010
	0	0	0	1	0	1	0	

Therefore, $10_{10} = 1010_2$.

c)

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
0								0
	0	0	0	0	0	0	0	

Therefore, $0_{10} = 0_2$.

Exercise 8

Convert the following to decimal.

a) 7_8 b) 11_8 c) 11027_8

Exercise 8 Solution

a)
$$\begin{aligned} 7_8 &= 7_8 \times 8^0 \\ &= 7 \times 1 \\ &= 7_{10} \end{aligned}$$

b)
$$\begin{aligned} 11_8 &= (1_8 \times 8^1) + (1_8 \times 8^0) \\ &= (1 \times 8) + (1 \times 1) \\ &= 8 + 1 \\ &= 9_{10} \end{aligned}$$

c) $$\begin{aligned} 11027_8 &= (1_8 \times 8^4) + (1_8 \times 8^3) + (0_8 \times 8^2) + (2_8 \times 8^1) + (7_8 \times 8^0) \\ &= (1 \times 4096) + (1 \times 512) + (0 \times 64) + (2 \times 8) + (7 \times 1) \\ &= 4096 + 512 + 0 + 16 + 7 \\ &= 4631_{10} \end{aligned}$$

Exercise 9

Convert the following to octal. Use the conversion table from Example 9.

a) 2_{10} b) 8_{10} c) 1089_{10}

Exercise 9 Solution

a)

Base 10	$8^5 = 32768$	$8^4 = 4096$	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$	Base 8
2						‖	2
	0	0	0	0	0	2	

Therefore, $2_{10} = 2_8$.

b)

Base 10	$8^5 = 32768$	$8^4 = 4096$	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$	Base 8
8					|		10
	0	0	0	0	1	0	

Therefore, $8_{10} = 10_8$.

c)

Base 10	$8^5 = 32768$	$8^4 = 4096$	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$	Base 8
1089			‖	|		|	2101
	0	0	2	1	0	1	

Therefore, $1089_{10} = 2101_8$.

Exercise 10

Convert the following to decimal.

a) 9_{16} b) $2F_{16}$ c) $1A2D_{16}$

Exercise 10 Solution

a)
$$\begin{aligned} 9_{16} &= 9_{16} \times 16^0 \\ &= 9 \times 1 \\ &= 9_{10} \end{aligned}$$

b)
$$\begin{aligned} 2F_{16} &= (2_{16} \times 16^1) + (F_{16} \times 16^0) \\ &= (2 \times 16^1) + (15 \times 16^0) \\ &= (2 \times 16) + (15 \times 1) \\ &= 32 + 15 \\ &= 47_{10} \end{aligned}$$

c)
$$\begin{aligned} 1A2D_{16} &= (1_{16} \times 16^3) + (A_{16} \times 16^2) + (2_{16} \times 16^1) + (D_{16} \times 16^0) \\ &= (1 \times 16^3) + (10 \times 16^2) + (2 \times 16^1) + (13 \times 16^0) \\ &= (1 \times 4096) + (10 \times 256) + (2 \times 16) + (13 \times 1) \\ &= 4096 + 2560 + 32 + 13 \\ &= 6701_{10} \end{aligned}$$

Exercise 11

Convert the following to hexadecimal. Use the conversion table from Example 11.

a) 11_{10} b) 17_{10} c) 3740_{10}

Exercise 11 Solution

a)

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
11				𝍸𝍸|	B
	0	0	0	B	

Therefore, $11_{10} = B_{16}$.

b)

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
17			|	|	11
	0	0	1	1	

Therefore, $17_{10} = 11_{16}$.

c)

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
3740		卌卌||||	卌||||	卌卌||	$E9C$
	0	E	9	C	

Therefore, $3740_{10} = E9C_{16}$.

Problem Set Solutions

When converting between two positional number systems, neither of which is the decimal system, convert to decimal first and then to the other number system from decimal.

1. Evaluate the following powers.

a) 5^3 b) 9^0 c) 8^2 d) 1^7 e) 16^1 f) 10^8

Problem 1 Solutions

a) 125 b) 1 c) 64 d) 1 e) 16 f) 100000000

2. Convert the following to decimal (base 10).

a) 5_8 b) D_{16} c) 1111_2 d) $A01F_{16}$ e) 10075_8 f) 1001110_2

Problem 2 Solutions

a)
$$\begin{aligned} 5_8 &= 5_8 \times 8^0 \\ &= 5 \times 1 \\ &= 5_{10} \end{aligned}$$

b)
$$\begin{aligned} D_{16} &= D_{16} \times 16^0 \\ &= 13 \times 16^0 \\ &= 13 \times 1 \\ &= 13_{10} \end{aligned}$$

c)
$$\begin{aligned} 1111_2 &= (1_2 \times 2^3) + (1_2 \times 2^2) + (1_2 \times 2^1) + (1_2 \times 2^0) \\ &= (1 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1) \\ &= 8 + 4 + 2 + 1 \\ &= 15_{10} \end{aligned}$$

d) $A01F_{16} = (A_{16} \times 16^3) + (0_{16} \times 16^2) + (1_{16} \times 16^1) + (F_{16} \times 16^0)$
$= (10 \times 16^3) + (0 \times 16^2) + (1 \times 16^1) + (15 \times 16^0)$
$= (10 \times 4096) + (0 \times 256) + (1 \times 16) + (15 \times 1)$
$= 40960 + 0 + 16 + 15$
$= 40991_{10}$

e) $10075_8 = (1_8 \times 8^4) + (0_8 \times 8^3) + (0_8 \times 8^2) + (7_8 \times 8^1) + (5_8 \times 8^0)$
$= (1 \times 4096) + (0 \times 512) + (0 \times 64) + (7 \times 8) + (5 \times 1)$
$= 4096 + 0 + 0 + 56 + 5$
$= 4157_{10}$

f) 1001110_2
$= (1_2 \times 2^6) + (0_2 \times 2^5) + (0_2 \times 2^4) + (1_2 \times 2^3) + (1_2 \times 2^2) + (1_2 \times 2^1) + (0_2 \times 2^0)$
$= (1 \times 64) + (0 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$
$= 64 + 0 + 0 + 8 + 4 + 2 + 0$
$= 78_{10}$

3. Convert the following to binary (base 2).
a) 111_{10} b) 7_8 c) $4E_{16}$

Problem 3 Solutions

a)

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
111	|	|		|	|	|	|	1101111
	1	1	0	1	1	1	1	

Therefore, $111_{10} = 1101111_2$.

b) First, convert 7_8 to decimal.

$7_8 = 7_8 \times 8^0$
$= 7 \times 1$
$= 7_{10}$

Then, convert to binary.

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
7					|	|	|	111
	0	0	0	0	1	1	1	

So, $7_{10} = 111_2$. And therefore, $7_8 = 111_2$.

c) First, convert $4E_{16}$ to decimal.

$$\begin{aligned} 4E_{16} &= (4_{16} \times 16^1) + (E_{16} \times 16^0) \\ &= (4 \times 16^1) + (14 \times 16^0) \\ &= (4 \times 16) + (14 \times 1) \\ &= 64 + 14 \\ &= 78_{10} \end{aligned}$$

Then, convert to binary.

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
78	|			|	|	|		1001110
	1	0	0	1	1	1	0	

So, $78_{10} = 1001110_2$. And therefore, $4E_{16} = 1001110_2$.

4. Convert the following to octal (base 8).

a) 83_{10} b) 101011_2 c) $3F_{16}$

Problem 4 Solutions

a)

Base 10	$8^4 = 4096$	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$	Base 8
83			|	||	|||	123
	0	0	1	2	3	

Therefore, $83_{10} = 123_8$.

b) First, convert 101011_2 to decimal.

$$\begin{aligned}101011_2 &= (1_2 \times 2^5) + (0_2 \times 2^4) + (1_2 \times 2^3) + (0_2 \times 2^2) + (1_2 \times 2^1) + (1_2 \times 2^0)\\ &= (1 \times 32) + (0 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)\\ &= 32 + 0 + 8 + 0 + 2 + 1\\ &= 43_{10}\end{aligned}$$

Then, convert to octal.

Base 10	$8^4 = 4096$	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$	Base 8
43				$\cancel{\vert\vert\vert\vert}$	$\vert\vert\vert$	53
	0	0	0	5	3	

So, $43_{10} = 53_8$. And therefore, $101011_2 = 53_8$.

c) First, convert $3F_{16}$ to decimal.

$$\begin{aligned}3F_{16} &= (3_{16} \times 16^1) + (F_{16} \times 16^0)\\ &= (3 \times 16) + (15 \times 1)\\ &= 48 + 15\\ &= 63_{10}\end{aligned}$$

Then, convert to octal.

Base 10	$8^4 = 4096$	$8^3 = 512$	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$	Base 8
63				$\cancel{\vert\vert\vert\vert}\ \vert\vert$	$\cancel{\vert\vert\vert\vert}\ \vert\vert$	77
	0	0	0	7	7	

So, $63_{10} = 77_8$. And therefore, $3F_{16} = 77_8$.

5. Convert the following to hexadecimal (base 16).

a) 267_{10} b) 1111110_2 c) 2075_8

Problem 5 Solutions

a)

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
267		$\vert$		$\cancel{\vert\vert\vert\vert}\ \cancel{\vert\vert\vert\vert}\ \vert$	$10B$
	0	1	0	B	

Therefore, $267_{10} = 10B_{16}$.

b) First, convert 1111110_2 to decimal.

1111110_2

$= (1_2 \times 2^6) + (1_2 \times 2^5) + (1_2 \times 2^4) + (1_2 \times 2^3) + (1_2 \times 2^2) + (1_2 \times 2^1) + (0_2 \times 2^0)$

$= (1 \times 64) + (1 \times 32) + (1 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$

$= 64 + 32 + 16 + 8 + 4 + 2 + 0$

$= 126_{10}$

Then, convert to hexadecimal.

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
126			~~IIII~~ II	~~IIII~~ ~~IIII~~ IIII	$7E$
	0	0	7	E	

So, $126_{10} = 7E_{16}$. And therefore, $1111110_2 = 7E_{16}$.

c) First, convert 2075_8 to decimal.

$2075_8 = (2_8 \times 8^3) + (0_8 \times 8^2) + (7_8 \times 8^1) + (5_8 \times 8^0)$

$= (2 \times 512) + (0 \times 64) + (7 \times 8) + (5 \times 1)$

$= 1024 + 0 + 56 + 5$

$= 1085_{10}$

Then, convert to hexadecimal.

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
1085		IIII	III	~~IIII~~ ~~IIII~~ III	$43D$
	0	4	3	D	

So, $1085_{10} = 43D_{16}$. And therefore, $2075_8 = 43D_{16}$.

6. Determine whether the following pairs of numbers are equal.

a) 10_{10} and 10_2 b) ABC_{16} and 2748_{10} c) 27_8 and 111_2 d) 1101110_2 and $6E_{16}$

Problem 6 Solutions

a) $$\begin{aligned} 10_2 &= (1_2 \times 2^1) + (0_2 \times 2^0) \\ &= (1 \times 2) + (0 \times 1) \\ &= 2 + 0 \\ &= 2_{10} \end{aligned}$$

Thus, $10_2 \neq 10_{10}$.

b) $$\begin{aligned} ABC_{16} &= (A_{16} \times 16^2) + (B_{16} \times 16^1) + (C_{16} \times 16^0) \\ &= (10 \times 16^2) + (11 \times 16^1) + (12 \times 16^0) \\ &= (10 \times 256) + (11 \times 16) + (12 \times 1) \\ &= 2560 + 176 + 12 \\ &= 2748_{10} \end{aligned}$$

Thus, $ABC_{16} = 2748_{10}$.

c) Neither of the given numbers are in decimal. There are two choices, either convert both to decimal and compare or convert one into decimal and then into the same as the other. This solution converts both into decimal.

$$\begin{aligned} 27_8 &= (2_8 \times 8^1) + (7_8 \times 8^0) \\ &= (2 \times 8) + (7 \times 1) \\ &= 16 + 7 \\ &= 23_{10} \end{aligned}$$

and

$$\begin{aligned} 111_2 &= (1_2 \times 2^2) + (1_2 \times 2^1) + (1_2 \times 2^0) \\ &= (1 \times 4) + (1 \times 2) + (1 \times 1) \\ &= 4 + 2 + 1 \\ &= 7_{10} \end{aligned}$$

Thus, $27_8 \neq 111_2$.

d) This solution converts both numbers into decimal and compares them.

1101110_2

$$\begin{aligned} &= (1_2 \times 2^6) + (1_2 \times 2^5) + (0_2 \times 2^4) + (1_2 \times 2^3) + (1_2 \times 2^2) + (1_2 \times 2^1) + (0_2 \times 2^0) \\ &= (1 \times 64) + (1 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1) \\ &= 64 + 32 + 0 + 8 + 4 + 2 + 0 \\ &= 110_{10} \end{aligned}$$

$$\begin{aligned}
\text{and} \qquad 6E_{16} &= (6_{16} \times 16^1) + (E_{16} \times 16^0)\\
&= (6 \times 16^1) + (14 \times 16^0)\\
&= (6 \times 16) + (14 \times 1)\\
&= 96 + 14\\
&= 110_{10}
\end{aligned}$$

Thus, $1101110_2 = 6E_{16}$.

7. How many symbols are used in a base n number system?

Problem 7 Solutions

By the definition of the base of a number system, the number of symbols is n.

8. We can define a base 3 positional number system whose symbols are $\bigcirc$, $\triangle$, and $\square$ using the following conversion chart.

Number System Symbols	$\bigcirc$	$\triangle$	$\square$
Decimal Values	0	1	2

The following exercises convert from the base 3 number system defined above to the decimal number system and vice versa. Recall that the methods used for both directions of conversion rely upon the base of the non-decimal number system, that is, the number of symbols.

a) Convert $\square\bigcirc\triangle\triangle\square_3$ to decimal.

b) Convert 104_{10} to the base 3 number system. Use the following conversion chart.

Base 10	$3^4 = 81$	$3^3 = 27$	$3^2 = 9$	$3^1 = 3$	$3^0 = 1$	Base 3

Problem 8 Solutions

a) To convert $\square\bigcirc\triangle\triangle\square_3$ to decimal, we write out the expansion.

$$\begin{aligned}
\square \bigcirc \triangle\triangle\square_3 &= (\square_3 \times 3^4) + (\bigcirc_3 \times 3^3) + (\triangle_3 \times 3^2) + (\triangle_3 \times 3^1) + (\square_3 \times 3^0) \\
&= (2 \times 3^4) + (0 \times 3^3) + (1 \times 3^2) + (1 \times 3^1) + (2 \times 3^0) \\
&= (2 \times 81) + (0 \times 27) + (1 \times 9) + (1 \times 3) + (2 \times 1) \\
&= 162 + 0 + 9 + 3 + 2 \\
&= 176
\end{aligned}$$

Thus, $\square \bigcirc \triangle\triangle\square_3 = 176_{10}$.

b) To convert 104_{10} to the base 3 number system, we will fill in the conversion table by writing a tally mark every time we subtract a power.

First, we note that $3^4 = 81$ is the largest power of 3 that is less than 104 and $104 - 81 = 23$. Then, $3^2 = 9$ is the largest power of 3 that is less than 23 and $23 - 9 = 14$. Then, $3^2 = 9$ is the largest power of 3 that is less than 14 and $14 - 9 = 5$. Then, $3^1 = 3$ is the largest power of 3 that is less than 5 and $5 - 3 = 2$. Then, $3^0 = 1$ is the largest power of 3 that is less than 2 and $2 - 2(1) = 0$.

Now, we fill in the bottom row by writing each symbol for the numeric value of the tallies for that column. The number on the bottom row, read from left to right, is the base 3 number equivalent to the decimal number. So, 104_{10} in the base 3 number system is $\triangle \bigcirc \square\triangle\square_3$.

Base 10	$3^4 = 81$	$3^3 = 27$	$3^2 = 9$	$3^1 = 3$	$3^0 = 1$	Base 3
104	|		||	|	||	$\triangle \bigcirc \square\triangle\square$
	$\triangle$	$\bigcirc$	$\square$	$\triangle$	$\square$	

9. As mentioned in the lesson, languages and words are a lot like number systems and numbers. In fact, the letters of the alphabet can be considered as the symbols of a base-26 number system.

Letters	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Decimal Values	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Letters	Q	R	S	T	U	V	W	X	Y	Z
Decimal Values	16	17	18	19	20	21	22	23	24	25

Create your own symbols (at least 4) for a positional number system (do not use digits or letters from the Latin alphabet). Create a conversion chart from your number system to the decimal system. Then, write your initials as a number in your number system. (That is, convert from the base 26 system defined above to the decimal system, and from the decimal system to your own system.)

Problem 9 Solutions

Solutions will vary. Here is an example using the initials CVB.

Start by creating a conversion chart using the new symbols.

Number System Symbols	$\bigstar$	$\heartsuit$	$\clubsuit$	$\diamondsuit$	$\spadesuit$
Decimal Values	0	1	2	3	4

Then, convert CVB_{26} to decimal.

$$\begin{aligned} CVB_{26} &= (C_{26} \times 26^2) + (V_{26} \times 26^1) + (B_{26} \times 26^0) \\ &= (2 \times 26^2) + (21 \times 26^1) + (1 \times 26^0) \\ &= (2 \times 676) + (21 \times 26) + (1 \times 1) \\ &= 1352 + 546 + 1 \\ &= 1899_{10} \end{aligned}$$

Finally, convert 1899_{10} to the base 5 number system defined above.

<table>
<tr><th>Base 10</th><th>$5^4 = 625$</th><th>$5^3 = 125$</th><th>$5^2 = 25$</th><th>$5^1 = 5$</th><th>$5^0 = 1$</th><th>Base 3</th></tr>
<tr><td rowspan="2">1899</td><td>|||</td><td></td><td></td><td>||||</td><td>||||</td><td rowspan="2">$\diamondsuit\bigstar\bigstar\spadesuit\spadesuit$</td></tr>
<tr><td>$\diamondsuit$</td><td>$\bigstar$</td><td>$\bigstar$</td><td>$\spadesuit$</td><td>$\spadesuit$</td></tr>
</table>

Thus, $CVB_{26} = \diamondsuit\bigstar\bigstar\spadesuit\spadesuit_5$.

10. The simplest non-positional number system is the unary number system. The unary system contains one symbol, the digit 1. The positive integer n is represented by repeating the symbol n times. Since there is only one symbol, position has no effect on its value.

    a) What is the base of the unary number system?

    b) What is an example of a unary system that is commonly used?

    c) The unary system is very simple and only requires one symbol. What is a disadvantage of this number system?

    d) Does there exist a number system with a smaller base than the unary system?

Problem 10 Solutions

a) Base 1.

b) Tally marks.

c) It takes a really long time to write out large numbers. Try writing out 143 using a unary number system.

d) No, there is no smaller base because you need at least one symbol in a number system.

11. A well-known non-positional number system is the Roman Number System. This system uses the following symbols from the Latin alphabet.

Roman Symbols	I	V	X	L	C	D	M
Decimal Values	1	5	10	50	100	500	1000

The values of the numbers depend on the relative positions between the symbols.

To convert from Roman numbers to decimal, start with the largest valued symbol and follow the two simplified rules below.

- Smaller valued symbols are subtracted from larger ones when they are written to the left.
- Smaller valued symbols are added to larger ones when they are written to the right.

Convert the following into decimal.

a) II b) IV c) VI d) XV e) IL f) CD g) $MMXXII$

Problem 11 Solutions

a) 2 b) 4 c) 6 d) 15 e) 49 f) 400 g) 2022

12. Consider a base n positional number system which uses the digits, in their typical order, as its first ten symbols.

 a) Convert 1_n into the decimal system.
 b) Convert 10_n into the decimal system.
 c) Convert 100_n into the decimal system.
 d) Convert 1000_n into the decimal system.
 e) Convert $(10^e)_n$ into the decimal system, where e is any whole number.

Problem 12 Solutions

a) $$\begin{aligned} 1_n &= 1_n \times n^0 \\ &= 1 \times 1 \\ &= 1_{10} \end{aligned}$$

b) $$\begin{aligned} 10_n &= (1_n \times n^1) + (0_n \times n^0) \\ &= (1 \times n) + (0 \times 1) \\ &= n + 0 \\ &= n_{10} \end{aligned}$$

c) $$\begin{aligned} 100_n &= (1_n \times n^2) + (0_n \times n^1) + (0_n \times n^0) \\ &= (1 \times n^2) + (0 \times n) + (0 \times 1) \\ &= n^2 + 0 + 0 \\ &= (n^2)_{10} \end{aligned}$$

d) $$\begin{aligned} 1000_n &= (1_n \times n^3) + (0_n \times n^2) + (0_n \times n^1) + (0_n \times n^0) \\ &= (1 \times n^3) + (0 \times n^2) + (0 \times n) + (0 \times 1) \\ &= n^3 + 0 + 0 + 0 \\ &= (n^3)_{10} \end{aligned}$$

e) Following the pattern, $(10^e)_n = (n^e)_{10}$.